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Tangent Bundle of globally symplectomorphic inhomogeneous Einstein manifold to R^{2n}

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Abstract:

In present paper study and discuss Calabi metric and tangent bundle. We study the concept of tangent bundle and its global symplectic coordinate in explicit manner for the Calabi's Kähler–Einstein metric which is inhomogeneous on the tubular and non-tubular domains.

Keywords: Symplectomorphism, Calabi's metric, Tangent Bundle

1 Introduction:

The Kahler form for an n dimensional manifold M_n defined by ϖ where the said manifold is the complex manifold which is also diffeomorphic to the R^{2n}. R^{2n} is almost congruent to C_n. Now lets define symplectic coordinates and they are admitted by M_n and ϖ then such coordinates are global symplectic coordinates. I manifold M_n is smooth and isomorphism is defined over it then such isomorphism is called diffeomorphism and defined as ¥: $M_n \rightarrow R^{2n}$. Such that diffeomorphism hold
¥*$\varpi_0 = \varpi$, where ϖ_0 is defined as $\sum_{k=1}^{n} dx_k \wedge dx_k$which is the symplectic form in standard manner defined over R^{2n}. Now our target is to obtain sufficient condition in context to complex structure or Riemannian structure of the involved manifold M_n. The said condition satisfy the existence of coordinates which are globally symplectic.

Bates L. [1] defines and study symplectic structure which subsequently study by Cuccu F. [4] and define properties when symplectic structure is globally on the complex manifold. Di Scala A. [3] further studied global symplectic structure and find results on the duality of the symplectic structure. Loi A. and Zuddas F. [2]. McDuff D. [5] studied Khaler manifold which is of non-positive curvature and further define symplectic structure, he also define and prove several theorem which define the global version of certain well defined theorems like Darboux theorem with sectional curvature which is non-positive. [3] Also define Bergman metric (R^{2n}, ϖ_0) including the concept of Jordan triple system.

2. Globally Symplectomorphism:

[1] Defined symplectic structure and [3] define symplectic coordinates globally. Symplectomorphism a symplectic map or is an isomorphism in the context of symplectic manifold. Explicit symplectic global coordinates in the sense of Calabi –Kahler-Einestien form ϖ on the domain

$$M_n = \frac{1}{2} R^n \oplus E_c \tag{2.1}$$

where $M_n \subseteq C^n$. $n \geq 2$ and $E_p \subset R^n$, so E_p is an open ball defined on R^n , center of which is at origin and a radius.

Theorem 2.1: Khaler manifold (M_n, ϖ) is globally symplectomorphic to (R^{2n}, ϖ)if it hold the mapping
$\S: M \rightarrow \mathfrak{R}^n \oplus i\mathfrak{R}^n \approx \mathfrak{R}^{2n}$, (u, v) → (gradient F, v) (2.2)
where F: $E_p \rightarrow \mathfrak{R}$, u = ($u_1$, u_n) → F(u) is the Khaler Potential for ϖ
that is if we define ϖ as partial derivative than gradient F = $\left(\frac{\partial F}{\partial u_1}, \frac{\partial F}{\partial u_2}, \frac{\partial F}{\partial u_3}, \ldots, \frac{\partial F}{\partial u_n}\right)$

Calabi E. [4] defines the isometric imbedding the complex manifold and in subsequent sections it gives formula for curvature tensor of (M_n, g), where g is the metric in connection to the defined Khaler form ϖ which is not homogenous in locally sense hence so is sectional curvature is negative that is not positive the g may or may not hold the assumptions of Mcduff theorem. If Mcduff theorem hold than the existence of globally symplectic coordinates. There is no explicit rule for computation of it.

For all Lagranginan manifold of (M_n, ϖ) we have certain rules and theorems to show the existence of symplectic geometry.

3. Calabi Metric

For a tubular domain which is complex $M_n = \frac{1}{2} R^n \oplus E_c$, where $M_n \subseteq C^n$. $n \geq 2$ and $E_p \subset R^n$, as defined above now we have a metric g defined on M_n with the condition $M_n \subseteq C^n$ and the defined Khaler form is provided as

$$\varpi = \frac{i}{2} \partial \check{\partial} (w_1 + \breve{w}_1, \ldots w_n + \breve{w}_n) \tag{3.1}$$

where F: $E_p \rightarrow \mathfrak{R}$, u = ($u_1$, u_n) → F(u), F is a radial function such that F(u_1, u_2, ...u_n) = G(s),
for s = $\left(\sum_{p=1}^{n} u_p^2\right)^{1/2}$, for all $u_m = \frac{w_m + \breve{w}_m}{2}$ and $v_m = \frac{w_m - \breve{w}_m}{2}$,
above condition satisfies the differential equation

$$\left(\frac{1}{s} Y'\right)^{n-1} Y'' = e^Y \tag{3.2}$$

For equation (3.2) the boundary conditions are $Y'(0) = 0, Y''(0) = Y^{\frac{y(0)}{n}}$ (3.3)

Calabi E. [4] discussed g Kahler metric which is smooth and complete and also does not contain the property of homogeneousness locally. Wolf J. [8] discussed the alternative and easier proof to show the completeness of the metric along with the condition of non homogeneity locally.

4. Tangent Bundle

For an abstract manifold M_n and over it defined vector field defines tangent vector T(t) for all t belongs to M_n. Therefore the defined field T over the M_n is the superset which contains all tangent vectors defined over M_n. This obtained set itself define differential structure on M_n. T as defined by the conditions mentioned is also contain the property of smoothness over the manifold M_n.

Definition 4.1

T the tangent bundle defined over manifold M_n is the union of the all tangents over M_n, that is,
$T_m = \bigcup_{p \in M} T_p M$.

Mapping $\mathfrak{Z}: T_p \rightarrow M_n$ which maps p to X, where X belongs to T_m for all p belongs to M_n then the mapping $\mathfrak{Z}$ is called projection.

Let $§: M \rightarrow \Re^n \oplus i\Re^n \approx \Re^{2n}$, (u, v) → (gradient F, v) is a smooth mapping over the manifold M_n.Now differential of mapping d§ maps as d§: $T_m \rightarrow T_{§(p)}R$, for each p belongs to M_n. the collection of these maps from T_m to $T_{§(p)}$ is gradient §.

If $§: M \rightarrow \Re^n \oplus i\Re^n \approx \Re^{2n}$ maps over open set $M \subset R^n$ smoothly into R^{2n}, the mapping d§: $T_m \rightarrow T_{§(p)}R$ is given by $d§(u, v) = \{§(u), \text{gradient } F(u)v\}$, which is a smooth mapping over the manifold.

Theorem 4.1

For an abstract manifold M_n the collection of mapping $d§: M \rightarrow \Re^n \oplus i\Re^n$, for all sectional § over the union of §, a differential structure over the manifold M_n on the tangent bundle T_p. $\mathfrak{Z}: T_p \rightarrow M_n$ is the mapping which is smooth defined over vector field on M_n if and only if it is smooth from M_n to T_p.

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